

# FRM financialriskmeter for Cryptos

Michael Althof

Vanessa Guarino

Rui Ren

Anna Shchekina

Humboldt-Universität zu Berlin [lvb.wiwi.hu-berlin.de](http://lvb.wiwi.hu-berlin.de) Charles University, WISE XMU, NCTU 玉山学者

#### Tail Events (TE)

- ⊡ TEs across Cryptos indicate increased risk
- ⊡ CoVaR measures joint TEs between 2 risk factors
- ⊡ CoVaR and other risk factors?
- ⊡ TENET Tail Event NETwork risk, Härdle Wang Yu (2017) J E'trics
- ⊡ FRM Financial Risk Meter for joint TEs



#### Risk Measures

- ⊡ VIX: IV based, does not reflect joint TEs
- ⊡ CoVaR concentrates on a pair of risk factors
- ⊡ CISS, Google trends, SRISK, …
- ⊡ FRM displays the full picture of TE dependencies
- ⊡ [Firamis.de/FRM](http://Firamis.de/frm) **hu.** [financialriskmeter](http://hu.berlin/FRM)

#### Call and Puts on BTCs

⊡ Listed at Bloomberg since 20200113

Prices from 20200221.1600 - 20200222.1100 Timestamps precise in the range 1E-3 sec. Calls, Puts with maturity 20200228









CRIX

**FRM** for Cryptos

FRM

VCRIX

#### **Outline**

- 1. Motivation  $\checkmark$
- 2. Genesis
- 3. Framework
- 4. Applications
- 5. Node influence metrics
- 6. Sensitivity analysis
- 7. Network centrality
- 8. Portfolio Construction
- 9. Conclusions



#### **VaR Value at Risk**

Probability measure based  $\Box$ 

$$
P(X_{i,t} \le VaR_{i,t}^{\tau}) \stackrel{\text{def}}{=} \tau, \quad \tau \in (0,1)
$$

 $\Box$   $X_{i,t}$  log return of risk factor (institution) *i* at *t* □ VaRs (0.99, 0.01) based on RMA, Delta Normal Method





#### <span id="page-7-0"></span>Quantiles and Expectiles

For r.v. Y obtain tail event measure: log returns  $q^{\tau}$  = arg min *θ*  $E \left\{ \rho_{\tau} (Y - \theta) \right\}$ 

asymmetric loss function

$$
\rho_{\tau}(u) = |u|^{c} |\tau - I_{\{u < 0\}}|
$$

 $c = 1$  ≻ quantiles  $c = 2$  > expectiles



**[Expectile as Quantile](#page-59-0)** 

8

#### Conditional Value at Risk

⊡ Adrian and Brunnermeier (2016) introduced CoVaR

$$
P\{X_{j,t} \leq Cov a R_{j|i,t}^{\tau} \mid X_{i,t} = VaR^{\tau}(X_{i,t}), M_{t-1}\} \stackrel{\text{def}}{=} \tau
$$

□  $M$ <sub>t-1</sub> vector of macro-related variables

 $\Box$  Nonlinear features,  $\tau = 0.05$ 



Figure: Goldman Sachs (Y), Citigroup (X), Confidence Bands, see Chao et al (2015)

FRM

#### CoVaR and the magic of joint TEs

⊡ CoVaR technique

$$
X_{i,t} = \alpha_i + \gamma_i^{\top} M_{t-1} + \varepsilon_{i,t}
$$
  
\n
$$
X_{j,t} = \alpha_{j|i} + \beta_{j|i} X_{i,t} + \gamma_{j|i}^{\top} M_{t-1} + \varepsilon_{j,t}
$$
  
\n
$$
F_{\varepsilon_{i,t}}^{-1}(\tau | M_{t-1}) = 0 \text{ and } F_{\varepsilon_{j,t}}^{-1}(\tau | M_{t-1}, X_{i,t}) = 0
$$
  
\n
$$
\widehat{VaR}_{i,t}^{\tau} = \widehat{\alpha}_i + \widehat{\gamma}_i^{\top} M_{t-1}
$$
  
\n
$$
\widehat{CoVaR}_{j|i,t}^{\tau} = \widehat{\alpha}_{j|i} + \widehat{\beta}_{j|i} \widehat{VaR}_{i,t}^{\tau} + \widehat{\gamma}_{j|i}^{\top} M_{t-1}
$$

CoVaR: First calculate VaRs, then compute the TE given a stressed risk factor.

FRM

#### <span id="page-10-0"></span>Linear Quantile Lasso Regression

$$
r_{j,t}^s = \alpha_{j,t}^s + A_{j,t}^{s\top} \beta_j^s + \varepsilon_{j,t}^s
$$
  
\n
$$
A_{j,t}^{s\top} \stackrel{\text{def}}{=} \left[ M_{t-1}^s, r_{-j,t}^s \right]
$$
 (1)

where:

 $\Gamma$   $\Gamma$ <sup>*s*</sup><sub>*-j*,*t*</sub> log returns of all cryptos except *j* ∈ 1:*J* at *t* ∈ 2:*T* 

- □ *s* length of moving window
- ⊡ log return of macro prudential variable at time *Ms <sup>t</sup>*−<sup>1</sup> *t* − 1
- $\Box$  For application, consider  $J = 15$ ,  $s = 63$



#### Lasso Quantile Regression

$$
\min_{\alpha_j^s, \beta_j^s} \left\{ n^{-1} \sum_{t=s}^{s+(n-1)} \rho_\tau (r_{j,t}^s - \alpha_j^s - A_{j,t}^{s\top} \beta_j^s) + \lambda_j^s \parallel \beta_j^s \parallel_1 \right\} \tag{2}
$$

- $\Box$  Check function  $\rho_{\tau}(u) = |u|^c |\tau I_{\{u < 0\}}|$  with  $c = 1, 2$ corresponding to quantile, expectile regression
	- ▶ *λ* creates size of "active set", i.e. spillover
	- ▶ *λ* is sensitive to residual size, i.e. TE size
	- ▶ reacts to singularity issues, i.e. joint TEs *λ*

### *λ* Role in Linear Lasso Regression

⊡ Osborne et al. (2000)

⊡ Dependence, time-varying, institution-specific

⊡ Size of model coefficients depends on,

$$
\lambda = \frac{\left(Y - X\beta(\lambda)\right)^{\top} X\beta(\lambda)}{\left\|\beta\right\|_{1}}
$$
 Coeff's depend on  $\lambda$ 

⊡ depends on: *λ*

- ▶ Residual size
- ▶ Condition of design matrix
- ▶ Active set

#### *λ* Role in Linear Quantile Regression

**□** *λ* size of estimated LQR coefficients Li Y, Zhu JL (2008)

$$
\lambda = \frac{(\alpha - \gamma)^{\top} X \beta(\lambda)}{\|\beta\|_1}
$$
 Coeff's  $(\lambda)$ 

$$
(\alpha - \gamma)^{\top} = \tau \, I_{\{Y - X\beta(\lambda) > 0\}} + (\tau - 1) \, I_{\{Y - X\beta(\lambda) < 0\}}
$$

⊡ Average penalty: indicator for tail risk,

$$
FRM^t \stackrel{def}{=} J^{-1} \sum_{j=1}^J \lambda_j^t
$$

⊡ The FRM time series is one index for joint TEs!

### *λ* Selection

⊡ Generalized approximate cross-validation (GACV) (Yuan, 2006)

min 
$$
GACV(\lambda_j^s)
$$
 = min  $\frac{\sum_{i=s}^{s+(n-1)} \rho_r(r_{j,t}^s - \alpha_j^s - A_{j,t}^{sT}\beta_j^s)}{n - df}$  (3)  
\nwhere: *df* dimensionality of fitted model  
\n $\square$   $\lambda$  as function of *j*, *t*  
\n $\square$  Distribution of  $\lambda$   
\n $\square$  Distribution of  $\lambda$   
\n $\square$  Differentizers



**FRM** for Cryptos

FRM@iTraxx

#### <span id="page-16-0"></span>FRM@Crypto Data

- ⊡ 15 largest cryptocurrencies
- ⊡ 6 macro related variables
- $□$  Quantile level  $τ = 0.05, 0.10, 0.25, 0.50$
- ⊡ Time window *s* = 63, 21
- ⊡ Time frame: 2014–2020
- ⊡ Macroeconomic risk factors:
	- ▶ US dollar index (average of USD vs main non-crypto currencies)
	- ▶ Yield level in USD (carry component for the drift)
	- ▶ VIX
	- ▶ CVIX (same as VIX, but on major fiat currencies)
	- ▶ S&P500

**FRM** for Cryptos

#### Methodology

- Obtain risk driver list of all historically active index members
- ⊡ Download daily rates in same currency (USD)
- □ Sort market cap decreasingly (to select *J* biggest risk drivers)
- ⊡ Calculate returns
- ⊡ On every trading day
	- ▶ Select *J* biggest risk driver's returns over *s* trading days
	- ▶ Attach returns of macroeconomic risk factors
	- $\blacktriangleright$  Calculate  $\lambda$  for all companies
	- $\blacktriangleright$  Calculate average  $\lambda$ , etc.
	- ▶ Store active set



#### FRM@Crypto Distribution



#### FRM@Crypto Distribution



#### FRM@Crypto Distribution



22

#### BTC and ETH dominate the market - FRM reflects?



#### BTC and ETH dominate the market - FRM reflects?



#### BTC and ETH dominate the market - FRM reflects?



#### Tail risk and window size sensitivity: FRM@Crypto Index



Figure: FRM@Crypto index for tail risk  $\tau = 5\%$ , 10%, 25%, 50% for  $s = 63$  (left) and  $s = 21$  (right). Data from 01 January 2020 to 17 May 2020.

**FRM** for Cryptos

#### Tail risk and window size sensitivity: CoStress



Table: Crypto currencies with high (top table) and low (bottom table) CoStress with the number of days they appeared in top/bottom 5 for tail risk  $\tau=5\%$ , 10%, 25%, 50%.

Data from 1 January 2020 to 17 May 2020.

#### FRM@Crypto Model Selection Methods



Table:  $\lambda_j$  and effective dimension of the *j*-fitted models, via solution path of the  $L1$ -norm QR algorithm

and formula for  $\tau = 5\%$ , 10%, 50%. In all the settings  $s = 63$ ,  $J = 15$ .

Data from 1 January 2020 to 17 May 2020.

#### Flight into Cash: 2018 vs 2020 Crises



**FRM** for Cryptos

#### FRM@Crypto Adjacency Matrix with Macro Variables

#### $= \tau = 0.05, 12$  February 2018



Few traditional macro variables explain crypto currency tail behaviour

FRN.

#### Visualising the Active Set: FRM@Crypto the Movie



20200804 FRM: 0.0226

Figure: Network analysis for FRM@Crypto from 4 August 2020 to 24 September 2020.

Size of the node corresponds to *λ*

#### FRM@Crypto Distribution under Covid Crisis



#### 29 April 2020 — Marginal Return Contribution to BTC





FRM

#### Types of Centrality of a Node

⊡ Degree centrality

- ▶ *In-degree* how many other coins affect the node
- ▶ *Out-degree* how many other coins the node affects
- ⊡ *Closeness* shortest path between the node and all other nodes
- ⊡ *Betweenness* the number of times a node acts as a bridge along the shortest path between two other nodes
- ⊡ *Eigenvector* takes into account that connections to highscoring nodes contribute more to the score of the node in question than equal connections to low-scoring nodes

#### FRM@Crypto Out-Degree Centrality



Left-hand side panel: # of outbounds links of BTC, ETH, XRP, BCH, BSV, LTC, EOS, BNB, XTZ, LIN, ADA, XLM, XMR, TRX, HT. Right-hand side panel: FRM index over time. Data from 01 March 2020 to 17 May 2020

FRM

#### FRM@Crypto In-Degree Centrality



Left-hand side panel: # of inbound links of BTC, ETH, XRP, BCH, BSV, LTC, EOS, BNB, XTZ, LIN, ADA, XLM, XMR, TRX, HT. Right-hand side panel: FRM index over time. Data from 01 March 2020 to 17 May 2020

FRM

#### FRM@Crypto Betweenness Centrality



Left-hand side panel: "bridge" behaviour measure for BTC, ETH, XRP, BCH, BSV, LTC, EOS, BNB, XTZ, LIN, ADA, XLM, XMR, TRX, HT. Right-hand side panel: FRM index over time. Data from 01 March 2020 to 17 May 2020

FRM

#### FRM@Crypto Closeness Centrality



Left-hand side panel: fastness in influencing of BTC, ETH, XRP, BCH, BSV, LTC, EOS, BNB, XTZ, LIN, ADA, XLM, XMR, TRX, HT. Right-hand side panel: FRM index over time. Data from 01 March 2020 to 17 May 2020

FRM

#### FRM@Crypto Eigenvector Centrality



Left-hand side panel: normalised eigenvector centrality of BTC, ETH, XRP, BCH, BSV, LTC, EOS, BNB, XTZ, LIN, ADA, XLM, XMR, TRX, HT. Right-hand side panel: FRM index over time. Data from 01 March 2020 to 17 May 2020

#### From Nodes to Network Centralisation

Extend the notion of *point centrality* on the entire network.

1. Average of all nodes  $>$  spirit of FRM

$$
C = \sum_{i=1}^{M} C(p_i)
$$

2. Freeman centralisation

$$
C = \frac{\sum_{i=1}^{M} [C(p_*) - C(p_i)]}{\max \sum_{i=1}^{M} [C(p_*) - C(p_i)]} = \frac{\sum_{i=1}^{M} [C(p_*) - C(p_i)]}{M^2 - 3M + 2}
$$

 $p_*$  is most central node, max is over all graphs with  $M$  nodes.

#### FRM@Crypto vs Average Degree Centrality



#### FRM@Crypto vs Average Betweenness Centrality



#### FRM@Crypto vs Average Closeness Centrality



#### FRM@Crypto vs Average Eigenvector Centrality



44

#### **Backtesting**

- ⊡ Assess model validity based on the usefulness of its predictions and not on the sophistication of the assumptions
- ⊡ How well the risk measured by individual lambdas or their average reflects the short-time riskiness of cryptos
	- ▶ Riskiness benchmark: rolling historical volatility
	- ▶ Estimation window: 63 days

#### FRM@Crypto Index and VCRIX



#### Graphical backtest, *τ* = 5 %



Figure: FRM@Crypto for  $\tau = 5\%$  and CRIX rolling variance 20150404-20200525

FRM

#### Graphical backtest,  $\tau = 10\%$



Figure: FRM@Crypto for  $\tau = 10\%$  and CRIX rolling variance 20150404-20200525

FRM

#### Graphical backtest, *τ* = 5 %



Figure: BTC lambda for  $\tau = 5\%$  and BTC rolling variance 20150201-20200428

FRM

#### Graphical backtest, *τ* = 10 %



Figure: BTC lambda for  $\tau = 10\%$  and BTC rolling variance 20150201-20200428

FRM

#### Graphical backtest, *τ* = 5 %



Figure: ETH lambdas for  $\tau = 5\%$  and ETH rolling variance 20151011-20200428

FRM

#### Graphical backtest,  $\tau = 10\%$



Figure: ETH lambdas for  $\tau = 10\%$  and ETH rolling variance 20151011-20200428

FRM

#### Crypto Returns and the Pricing Kernel

According to the basic pricing equation

$$
E_t^{\mathbb{P}}[m_{t+1}R_{i,t+1}] = 1
$$
 (4)

 $m_t$  *marginal rate of substitution,*  $R_{i,t}$  *return of*  $i$ *-th crypto.* 

Considering log returns  $r_{i,t} = \log(R_{i,t}) \approx R_{i,t} - 1$ 

$$
E_t^{\mathbb{P}}[(1 + r_{i,t+1})m_{t+1}] \approx 1
$$
 (5)

Substituting in (4) risk-free rate  $R_{i,t} = R^f$ 

$$
\mathbf{E}_t^{\mathbb{P}}[m_{t+1}] = 1 \tag{6}
$$

#### Link to Sharpe Ratio

Combining (5) and (6)

$$
\mathbf{E}_t^{\mathbb{P}}[m_{t+1}r_{i,t+1}] \approx 0 \tag{7}
$$

Hence, the "Sharpe" ratio of  $r_{i,t}$  is bounded by  $\sigma(m_t)$ 

$$
(7) \iff E_t^{\mathbb{P}}[m_{t+1}] E_t^{\mathbb{P}}[r_{i,t+1}] + \text{Cov}_t^{\mathbb{P}}[m_{t+1}r_{i,t+1}] \approx 0
$$
  
\n
$$
\iff E_t^{\mathbb{P}}[r_{i,t+1}] \approx -\text{Corr}_t^{\mathbb{P}}[m_{t+1}r_{i,t+1}] \sigma(m_{t+1}) \sigma(r_{i,t+1})
$$
  
\n
$$
\iff \left| E_t^{\mathbb{P}}[r_{i,t+1}] \right| \leq \sigma(m_{t+1}) \sigma(r_{i,t+1})
$$
  
\n(8)

#### Role of Lambda as Penalisation Parameter

An analogous inequality to (8) holds for the empirical distribution

$$
\left| \widehat{\mathbf{E}}_t^{\mathbb{P}}[r_{i,t+1}] \right| \leq \sigma(\widehat{m}_{t+1}) \widehat{\sigma}(r_{i,t+1})
$$

$$
> \left| \mathbf{E}_{t}^{\mathbb{P}}[r_{i,t+1}] - \left| \widehat{\mathbf{E}}_{t}^{\mathbb{P}}[r_{i,t+1}] \right| \leq \sigma(m_{t+1}) \sigma(r_{i,t+1}) - \sigma(\widehat{m}_{t+1}) \widehat{\sigma}(r_{i,t+1}) \tag{9}
$$

Due to persistency of volatility of returns  $\hat{\sigma}(r_{i,t+1}) \approx \sigma(r_{i,t+1})$ 

 $\lambda_{i,t}$  chosen with CV tries to minimise the LHS  $\ge$ 

$$
\lambda_{i,t} \propto \sigma(\widehat{m}_{t+1}) - \sigma(m_{t+1})
$$

#### FRM in FinTech, Cryptos, …



#### Vol 1. 2019 on Crypto Currencies **of and response to IL-2** V. Holán · M. Lipoldová · P. Demant **27**





EDITORS: **Wolfgang Karl Härdle and Steven Kou**

# **Digital Finance**

Volume 1 • Number 1 • January 2019 • pp. 1– xx

Volume 1 - Number 1 - January 2019 - pp. 1-xx

 $\mathfrak{D}$  Springer

**Smart Data Analytics, Investment Innovation, and Financial Technology**



FRM



Michael Althof Vanessa Guarino Wolfgang K Härdle Rui REN Anna Shchekina



Alla Petukhina Ang LI Souhir Ben Amor Alex Truesdale Ilyas Agakishiev



**FRM** for Cryptos

#### **References**

- ⊡ Adrian J, Brunnermeier M (2016) CoVaR, American Economic Review, 106 (7): 1705-41, [DOI: 10.1257/aer.20120555](https://www.aeaweb.org/articles?id=10.1257/aer.20120555)
- ⊡ Buraschi A, Corielle F (2005). Risk management of time-inconsistency: Model updating and recalibration of no-arbitrage models. J Banking and Finance 29: 2883–907
- ⊡ Chao SK, Härdle WK, Wang W (2015) Quantile Regression in Risk Calibration. Handbook for Financial Econometrics and Statistics, Cheng-Few Lee, ed., Springer Verlag, DOI: 10.1007/978-1-4614-7750-1\_54.
- ⊡ Härdle WK, Wang W, Zbonakova L (2018) Time Varying Lasso, in Applied Quantitative Finance 3rd ed, (Chen, Härdle, Overbeck eds.) Springer Verlag, ISBN 978-3-662-54486-0
- ⊡ Keilbar G (2018) Modeling systemic risk using Neural Network Quantile Regression, [MSc thesis](https://edoc.hu-berlin.de/bitstream/handle/18452/20079/master_keilbar_georg.pdf?sequence=3&isAllowed=y)
- ⊡ Li Y, Zhu JL (2008) L1 Norm Quantile Regression, J Comp Graphical Statistics 17(1): 1-23
- ⊡ Osborne MR, Presnell B, Turlach BA (200) J Comp Graphical Statistics Vol. 9, 319-337
- ⊡ Yuan, M. (2006), GACV for Quantile Smoothing Splines, Computational Statistics & Data Analysis, 50: 813{829



# FRM financialriskmeter for Cryptos

Michael Althof

Vanessa Guarino

Rui Ren

Anna Shchekina

Humboldt-Universität zu Berlin [lvb.wiwi.hu-berlin.de](http://lvb.wiwi.hu-berlin.de) Charles University, WISE XMU, NCTU 玉山学者

#### <span id="page-59-0"></span>Expectile as Quantile

 $e_{\tau}(Y)$  is the  $\tau$ -quantile of the cdf  $T$ , where

$$
T(y) = \frac{G(y) - x F(y)}{2\{G(y) - y F(y)\} + y - \mu_Y}
$$

and

$$
G(y) = \int_{-\infty}^{y} u \, dF(u)
$$



**FRM** for Cryptos

#### <span id="page-60-0"></span>Cryptocurrencies List (as per 24 May 2020)



Source: [www.coingecko.com](https://www.coingecko.com/en)

